

PHYSICS 225 A - HOMEWORK ~~4~~ 4

1

1

(a) σ spinless, all angular momentum orbital

$$\text{Parity: } (-1)^L \Rightarrow J^P = 0^+, 1^-, 2^+$$

This does not explain quantum numbers of actual mesons, eg π is 0^-

(b) $P(\sigma, V_q) = (-1)(-1)^L = (-1)^{L+1}$

$$L=0 \rightarrow J=1 \rightarrow 1^-$$

$$L=1 \rightarrow J=0, 1, 2 \rightarrow 0^+, 1^+, 2^+, \dots$$

$$L=2 \rightarrow J=1, 2, 3 \rightarrow 1^-, 2^-, 3^+, \dots$$

Inconsistent with pseudoscalar meson quantum numbers

2

(a) $3 \otimes 3 = 6 \oplus \bar{3}$ (Halzen and Martin, Figure 2.6)

(b) 6 is flavor symmetric, one isospin triplet
one isospin doublet
one isospin singlet



$\bar{3}$ is flavor antisymmetric, one isospin doublet
one isospin singlet



(c) Lowest mass, $L=0 \Rightarrow P = (-1)^L = +1$

$$|\psi\rangle = |\text{spin}\rangle |\text{space}\rangle |\text{flavor}\rangle |\text{color}\rangle$$

↑
symmetric
for $L=0$

↑
antisymmetric

$$S=1 \text{ symmetric} \Rightarrow 6 \text{ representation, } J^P = 1^+$$

$$S=0 \text{ antisymmetric} \Rightarrow \bar{3} \text{ representation, } J^P = 0^+$$

③ Halzen and Martin 2.12

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

The 10 is fully symmetric $|10\rangle_{uds} = \frac{1}{\sqrt{6}} (uds + usd + dus + sud + sdu + dsu)$

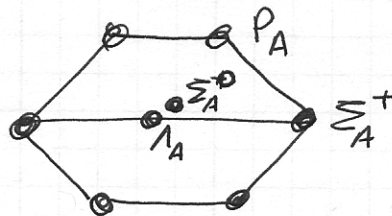
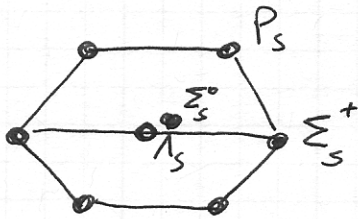
The 1 is totally antisymmetric

$$|1\rangle_{uds} = \frac{1}{\sqrt{6}} (uds - usd - dus - sdu + sud + dsu)$$

This must be because (a) All flavors must appear with equal footings

(b) Acting with $I_{\pm}, U_{\pm}, V_{\pm}$ must always get zero.

There are four more uds states belonging to the two 8

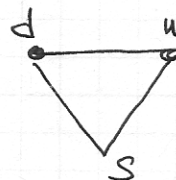
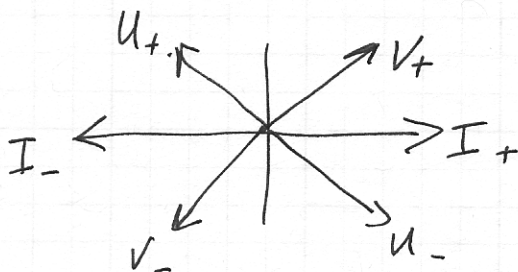


"S" and "A" refer to symmetry/antisymmetry under interchange of $1 \leftrightarrow 2$

$$P_S = \frac{1}{\sqrt{6}} [(ud+du)u - 2uud] \quad \text{Halzen \& Martin 2.62}$$

$$P_A = \frac{1}{\sqrt{2}} [ud - du]u \quad \text{Halzen \& Martin 2.60}$$

Raising and lowering operators



This is also obvious because it is the only combination antisymmetric in $1 \leftrightarrow 2$ (3)

$$U-P_A \sim \Sigma_A^+ \quad \left(\text{dropping multiplicative constants -} \right. \\ \left. \text{I will restore normalization at the end} \right)$$

$$\Sigma_A^+ \sim (us-su)u \quad \Sigma_A^0 = I_- \Sigma_A^+ \sim (ds-sd)u + (us-su)d$$

$$\Sigma_A^0 = \frac{1}{2} (dsu-sdu+usd-sud) \quad \left(\text{factor of } \frac{1}{2} \text{ from normalization} \right)$$

The singlet state Λ_A^0 must be (1) antisym with respect to $1 \leftrightarrow 2$
(2) Orthogonal to Σ_A^0 & $|1\rangle_{uds}$

$$\text{I can write } |1\rangle_{uds} = \frac{1}{\sqrt{6}} \{ [ds-sd]u + [su-us]d + [ud-du]s \}$$

$$\Sigma_A^0 = \frac{1}{2} \{ [ds-sd]u + [us-su]d \}$$

Clearly Σ_A^0 and $|1\rangle_{uds}$ are orthogonal

Λ_A^0 will be something like

$$\Lambda_A^0 \sim a [ds-sd]u + b [us-su]d + c [ud-du]s$$

In order for Λ_A^0 to be orthogonal to Σ_A^0 , $a = -b$

$$\Rightarrow \Lambda_A^0 \sim a [ds-sd]u + a [su-us]d + c [ud-du]s$$

$$\text{But } \langle 1 | \Lambda_A^0 \rangle = \frac{1}{\sqrt{6}} (a+a+c) = 0 \Rightarrow c = -2a$$

And to get the normalization right

$$\Lambda_A^0 = \frac{1}{\sqrt{12}} \{ [ds-sd]u + [su-us]d - 2[ud-du]s \}$$

$$\text{Also } U-P_S \sim \Sigma_S^+, P_S \sim (ud+du)u - 2und$$

$$\Sigma_S^+ \sim (us+su)u - 2uus \quad \Sigma_S^0 = I_- \Sigma_S^+ \sim (ds+sd)u + (us+su)d$$

$$\Sigma_s^0 \sim I - \Sigma_s^+ \sim (ds+sd)u + (us+su)d - 2uds - 2dus$$

Restoring the normalization

$$\Sigma_s^0 = \frac{1}{\sqrt{12}} \{ [ds+sd]u + [us+su]d - 2[ud+du]s \}$$

Λ_s^0 must be

- symmetric $1 \leftrightarrow 2$
- orthogonal to Σ_s^0
- orthogonal to $|10\rangle_{uds}$

Write $\Lambda_s^0 \sim a[ds+sd]u + b[us+su]d + c[ud+du]s$

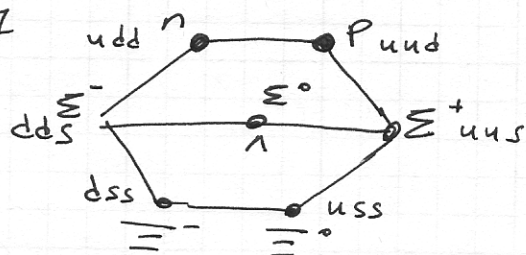
$\langle \Sigma_s^0 | \Lambda_s^0 \rangle = 0 \Rightarrow a+b-2c=0$

$\langle 10 | \Lambda_s^0 \rangle = 0 \Rightarrow a+b+c=0$

Solution is $c=0, a=-b$

$$\Lambda_s^0 = \frac{1}{2} \{ [ds-sd]u - [us+su]d \}$$

④ Helzen and Martin 2.17



Σ^+ : Its wavefunction can be obtained from the proton with the replacement $d \rightarrow s$

$\Rightarrow$ Since $\mu_p = \frac{1}{3} (4\mu_u - \mu_d)$

$$\mu_{\Sigma^+} = \frac{1}{3} (4\mu_u - \mu_s)$$

Σ^- : Same as neutron, but with $u \rightarrow s$

$$\mu_{\Sigma^-} = \frac{1}{3} (4\mu_d - \mu_s)$$

Ξ^0 : Same as proton, but with $u \rightarrow s, d \rightarrow u$

$$\mu_{\Xi^0} = \frac{1}{3} (4\mu_s - \mu_u)$$

Ξ^- : Same as Σ^- , but with $s \rightarrow d, d \rightarrow s$

$$\mu_{\Xi^-} = \frac{1}{3} (4\mu_s - \mu_d)$$

We can obtain the Σ^0 wavefunction from the Σ^+ (obtained from the proton with $d \rightarrow s$) and applying I -

$$|\Sigma^+ \uparrow\rangle = \sqrt{\frac{1}{18}} [u \uparrow u \downarrow s \uparrow + u \downarrow u \uparrow s \uparrow - 2u \uparrow u \uparrow s \downarrow + \text{permutations}]$$

$$|\Sigma^0 \uparrow\rangle = \sqrt{\frac{1}{36}} [d \uparrow u \downarrow s \uparrow + u \uparrow d \downarrow s \uparrow + d \downarrow u \uparrow s \uparrow + u \downarrow d \uparrow s \uparrow - 2d \uparrow u \uparrow s \downarrow - 2u \uparrow d \uparrow s \downarrow + \text{permutations}]$$

$$\mu_{\Sigma^0} = \frac{1}{36} [(\mu_d + \mu_s - \mu_u) + (\mu_u - \mu_d + \mu_s) + (-\mu_d + \mu_u + \mu_s) + (-\mu_u + \mu_d + \mu_s) + 4(\mu_d + \mu_u - \mu_s) + 4(\mu_d + \mu_u + \mu_s)] \times 3$$

$$\mu_{\Sigma^0} = \frac{1}{12} [\mu_d (1-1-1+1+4+4) + \mu_s (1+1+1+1-4-4) + \mu_u (-1+1-1+4+4)] = \frac{1}{12} [8\mu_d + 8\mu_u - 4\mu_s]$$

$$\boxed{\mu_{\Sigma^0} = \frac{1}{3} [2\mu_u + 2\mu_d - \mu_s]}$$

The Λ is an isospin singlet. Symmetry considerations tell us that we can only write it as

$$|\Lambda \uparrow\rangle = \frac{1}{\sqrt{12}} [(u \uparrow d \downarrow - d \uparrow u \downarrow) s \uparrow - (u \downarrow d \uparrow - d \downarrow u \uparrow) s \uparrow + \text{permutations}]$$

$$\mu_{\Lambda} = \frac{3}{12} [(\mu_u - \mu_d + \mu_s) + (\mu_d - \mu_u + \mu_s) + (-\mu_u + \mu_d + \mu_s) + (-\mu_d + \mu_u + \mu_s)]$$

$$\mu_{\Lambda} = \frac{1}{4} [\mu_u (1-1-1+1) + \mu_d (-1+1+1-1) + \mu_s (1+1+1+1)]$$

$$\boxed{\mu_{\Lambda} = \mu_s}$$

Now numerically, assuming perfect SU(3)

$$\mu_u = -2\mu_d = -2\mu_s$$

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d) = \frac{1}{3}(-8\mu_d - \mu_d) = -3\mu_d$$

Also $\mu_p = 2.79 \mu_N \Rightarrow \boxed{\mu_d = -0.93 \mu_N}$

$\uparrow$ DATA $\uparrow$ nuclear magneton

Incidentally: $\mu_d = -\frac{1}{3} \frac{1}{2m_d} = -\frac{1}{6m_d}$ $\mu_N = \frac{1}{2m_N}$

$\uparrow$ nucleon mass

Then $-\frac{1}{6m_d} = -0.93 \mu_N$ gives $\underline{m_d \approx 0.36 m_N}$

BARYON	CALCULATED μ	SU(3) APPROX	DATA
p	$\frac{1}{3}(4\mu_u - \mu_d) \longrightarrow$	$-3\mu_d = 2.79 \mu_N$	(INPUT)
n	$\frac{1}{3}(4\mu_d - \mu_u) \longrightarrow$	$2\mu_d = -1.86 \mu_N$	$-1.91 \mu_N$
Σ^+	$\frac{1}{3}(4\mu_u - \mu_s) \longrightarrow$	$-3\mu_d = 2.79 \mu_N$	$2.46 \mu_N$
Σ^-	$\frac{1}{3}(4\mu_d - \mu_s) \longrightarrow$	$\mu_d = -0.93 \mu_N$	$-1.16 \mu_N$
Σ^0	$\frac{1}{3}(2\mu_u + 2\mu_d - \mu_s) \longrightarrow$	$-\mu_d = +0.93 \mu_N$	?
Ξ^-	$\frac{1}{3}(4\mu_s - \mu_d) \longrightarrow$	$\mu_d = -0.93 \mu_N$	$-0.65 \mu_N$
Ξ^0	$\frac{1}{3}(4\mu_s - \mu_u) \longrightarrow$	$2\mu_d = -1.86 \mu_N$	$-1.25 \mu_N$
Λ	$\mu_s \longrightarrow$	$\mu_d = -0.93 \mu_N$	$-0.61 \mu_N$

5) Halzen & Martin 2.18

$$|p \uparrow\rangle = \frac{1}{\sqrt{2}} [P_A \chi(M_S) - P_S \chi(M_A)]$$

$$P_A = \frac{1}{\sqrt{2}} (ud - du)u \quad P_S = \frac{1}{\sqrt{6}} [(ud + du)u - 2uud]$$

$$\chi(M_A) = \sqrt{\frac{1}{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \quad \chi(M_S) = \sqrt{\frac{1}{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)$$

$$|p \uparrow\rangle = \frac{1}{\sqrt{24}} [(udu - duu) (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)] + \\ - \frac{1}{\sqrt{24}} [(udu + duu - 2uud) (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)]$$

$$|p \uparrow\rangle = \frac{1}{\sqrt{24}} [udu (2\downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) + duu (\cancel{2\uparrow\uparrow\downarrow} - 2\uparrow\downarrow\uparrow) \\ + 2uud (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)]$$

$$\mu_p = \frac{1}{24} [4\mu_d + 4\mu_d + 4\mu_d + 4\mu_d + 4\mu_d + 4\mu_d] = \mu_d$$

For the neutron, I just replace $u \leftrightarrow d \Rightarrow \mu_n = \mu_u$

$$\text{So } \frac{\mu_n}{\mu_p} = \frac{\mu_u}{\mu_d} \quad \text{but } \mu_u = -2\mu_d \Rightarrow \boxed{\frac{\mu_n}{\mu_p} = -2}$$

6) Halzen and Martin 2.30

$$\vec{S}_i = \frac{1}{2} \vec{\sigma}_i \quad \vec{\sigma}_1 + \vec{\sigma}_2 = 2(\vec{S}_1 + \vec{S}_2) = 2\vec{S}_{\text{tot}}$$

$$(\vec{\sigma}_1 + \vec{\sigma}_2)^2 = \sigma_1^2 + \sigma_2^2 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 4S_{\text{tot}}^2 \quad (1)$$

$$S_1^2 = S_2^2 = \frac{1}{2} (1 + \frac{1}{2}) = \frac{3}{4} \quad S_{\text{tot}}^2 = S(S+1)$$

$$\text{Also } \vec{\sigma}_1 = \frac{1}{2} \vec{S}_1 \Rightarrow \sigma_1^2 = 3 \quad \text{and} \quad \sigma_2^2 = 3$$

Equation (1) becomes

$$3 + 3 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 4S(S+1)$$

$$\vec{\sigma}_1 \vec{\sigma}_2 = 2s(s+1) - 3$$

For the π $m_\pi = m_u + m_d - \frac{3a}{m_u m_d}$ since $S=0$

For the K^* $m_{K^*} = m_u + m_s + \frac{a}{m_u m_s}$ since $S=1$

Then, in units of $(\text{GeV})^2$ $a = 0.16 m_u^2 = 0.0154$

η_c : $2m_c - \frac{a}{m_c^2} = 2 \cdot 1.65 - \frac{3 \cdot 0.0154}{(1.65)^2} = \underline{3.29 \text{ GeV}}$ expt 2.98 GeV

J/ψ : $2m_c + \frac{a}{m_c^2} = 2 \cdot 1.65 + \frac{0.0154}{(1.65)^2} = \underline{3.31 \text{ GeV}}$ expt 3.10 GeV

D : $m_c + m_u - \frac{a}{m_c m_u} = 1.65 + 0.31 - \frac{3 \cdot 0.0154}{(1.65)(0.31)} = \underline{1.87 \text{ GeV}}$ expt 1.87 GeV

$D_s(F)$: $m_c + m_s - \frac{3a}{m_c m_s} = 1.65 + 0.48 - \frac{3 \cdot 0.0154}{(1.65)(0.48)} = \underline{2.07 \text{ GeV}}$ expt 1.97 GeV

D^* : $m_c + m_u + \frac{a}{m_c m_u} = \underline{1.99 \text{ GeV}}$ expt 2.01 GeV

$D_s^*(F^*)$: $m_c + m_s + \frac{a}{m_c m_s} = \underline{2.15 \text{ GeV}}$ expt 2.11 GeV

π : $m_u + m_d - \frac{3a}{m_u m_d} = \underline{0.14 \text{ GeV}}$ expt 0.14 GeV

ρ : $m_u + m_d + \frac{a}{m_u m_d} = \underline{0.78 \text{ GeV}}$ expt 0.77 GeV

K : $m_u + m_s - \frac{3a}{m_u m_s} = \underline{0.48 \text{ GeV}}$ expt 0.49 GeV

K^* : $m_u + m_s + \frac{a}{m_u m_s} = \underline{0.89 \text{ GeV}}$ expt 0.89 GeV

ϕ : $2m_s + \frac{a}{m_s^2} = \underline{1.03 \text{ GeV}}$ expt 1.02 GeV

Then

$$m_p - m_\pi = \underline{\underline{0.63 \text{ GeV}}}$$

$$\frac{m_s}{m_u} [m(K^*) - m(K)] = \underline{\underline{0.62 \text{ GeV}}}$$

$$\frac{m_c}{m_u} [m(D^*) - m(D)] = \underline{\underline{0.75 \text{ GeV}}}$$

$$\frac{m_c m_s}{m_u^2} [m(D_s^*) - m(D_s)] = \underline{\underline{1.15 \text{ GeV}}}$$

7 Helzen & Martin 2.31

$$\vec{S}_i = \frac{1}{2} \vec{\sigma}_i$$

$$(\vec{\sigma}_1 + \vec{\sigma}_2 + \vec{\sigma}_3)^2 = 4 S_{\text{TOT}}^2$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2 \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j = 4 S_{\text{TOT}}^2$$

$$\sigma_i^2 = 3 \quad (\text{see previous problem})$$

$$9 + 2 \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j = 4 S_{\text{TOT}}^2, \quad \underline{\underline{\sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j = 2 S_{\text{TOT}}^2 - \frac{9}{2}}}}$$

$$\text{For the } \Delta, S_{\text{TOT}}^2 = \frac{3}{2} \left(\frac{3}{2} + 1 \right) = \frac{15}{4}$$

$$\text{For the } N, S_{\text{TOT}}^2 = \frac{1}{2} \left(\frac{1}{2} + 1 \right) = \frac{3}{4}$$

Then, using $m_u = m_d = m$

$$m(\Delta) = 3m + \frac{q'}{2m^2} \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j = 3m + \frac{q'}{2m^2} \left[\frac{15}{2} - \frac{9}{2} \right] = 3m + \frac{3q'}{2m^2}$$

$$m(N) = 3m + \frac{q'}{2m^2} \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j = 3m + \frac{q'}{2m^2} \left[\frac{3}{2} - \frac{9}{2} \right] = 3m - \frac{3q'}{2m^2}$$

$$m(\Delta) - m(N) = \frac{6q'}{2m^2} = \frac{3q'}{m^2}$$

$$\text{For the } \pi, \text{ previous problem } m(\pi) = 2m - \frac{3q}{m^2}$$

$$\text{For the } p, \text{ previous problem } m(p) = 2m + \frac{q}{m^2}$$

$\Rightarrow$ if $q' = q$ we get ~~ref~~.

$$m(p) - m(\pi) = \frac{4e'}{m^2}, \text{ or } m(\Delta) - m(N) = \frac{3}{4} [m(p) - m(\pi)]$$

⑧ Halzen and Martin 2.32

Σ^+ has $I=1$ $I_3=+1$ $\Sigma^+ \sim uus$

Σ^- has $I=1$ $I_3=-1$ $\Sigma^- \sim dds$

Σ^0 completes the triplet $\Sigma^0 \sim \frac{(ud+du)}{\sqrt{2}}s$

Λ^0 must then be $\Lambda^0 \sim \frac{(ud-du)}{\sqrt{2}}s$

Ground state, $L=0 \Rightarrow$ qq space wavefunction symmetric
Color wavefunction asymmetric
Total wavefunction must be asymmetric

For Σ , qq flavor symmetric $\Rightarrow$ Spin symmetric $\Rightarrow S_{qq}=1$

For Λ^0 , qq flavor antisymmetric $\Rightarrow$ Spin antisymmetric $\Rightarrow S_{qq}=0$

$$S_{\text{TOT}}^2 = (\vec{S}_1 + \vec{S}_2)^2 \Rightarrow \vec{S}_1 \vec{S}_2 = \frac{1}{2} (S_{\text{TOT}}^2 - S_1^2 - S_2^2)$$

$$\text{For } \Sigma : \vec{S}_1 \vec{S}_2 = \frac{1}{2} [1(1+1) - \frac{1}{2}(\frac{1}{2}+1) - \frac{1}{2}(\frac{1}{2}+1)] = \frac{1}{4}$$

$$\text{For } \Lambda : \vec{S}_1 \vec{S}_2 = \frac{1}{2} [0(0+1) - \frac{1}{2}(\frac{1}{2}+1) - \frac{1}{2}(\frac{1}{2}+1)] = -\frac{3}{4}$$

$$\text{Now consider } \vec{J}^2 = (\vec{S}_1 + \vec{S}_2 + \vec{S}_3)^2 = (\vec{S}_{\text{TOT}} + \vec{S}_3)^2$$

$$\vec{J}^2 = S_{\text{TOT}}^2 + S_3^2 + 2 \vec{S}_{\text{TOT}} \vec{S}_3$$

$$\text{So } \vec{S}_{\text{TOT}} \vec{S}_3 = (\vec{S}_1 + \vec{S}_2) \vec{S}_3 = \frac{1}{2} [J^2 - S_{\text{TOT}}^2 - S_3^2]$$

$$\Lambda^0 : J=\frac{1}{2} \quad S_{\text{TOT}}=0 \quad S_3=\frac{1}{2} \Rightarrow (\vec{S}_1 + \vec{S}_2) \vec{S}_3 = 0 \quad \checkmark$$

$$\Sigma : J=\frac{1}{2} \quad S_{\text{TOT}}=1 \quad S_3=\frac{1}{2} \Rightarrow (\vec{S}_1 + \vec{S}_2) \vec{S}_3 = -1 \quad \checkmark$$

$$\Sigma^* : J=\frac{3}{2} \quad S_{\text{TOT}}=1 \quad S_3=\frac{1}{2} \Rightarrow (\vec{S}_1 + \vec{S}_2) \vec{S}_3 = +\frac{1}{2} \quad \checkmark$$

Equation (2.92), using $m_u = m_d$ and $\vec{\tau}_i \cdot \vec{\tau}_j = 4 \vec{S}_i \cdot \vec{S}_j$
(And using the result from H&M 2.30)

$$m(\Lambda_Q) = 2m_u + m_q + \frac{a'}{2} \left[-\frac{3}{m_u^2} + \frac{0}{m_u m_q} \right] = m_0 - \frac{3a'}{2m_u^2} \quad \checkmark$$

$$m(\Sigma_Q) = 2m_u + m_q + \frac{a'}{2} \left[\frac{1}{m_u^2} + \frac{(-4)}{m_u m_q} \right] = m_0 + \frac{2a'}{m_u^2} \left[\frac{1}{4} - \frac{m_u}{m_q} \right] \quad \checkmark$$

$$m(\Sigma_Q^*) = 2m_u + m_q + \frac{a'}{2} \left[\frac{1}{m_u^2} + \frac{2}{m_u m_q} \right] = m_0 + \frac{a'}{m_u^2} \left[\frac{1}{2} + \frac{m_u}{m_q} \right] \quad \checkmark$$

$$\text{Then } m(\Sigma_c) - m(\Lambda_c) = \frac{2a'}{4m_u^2} - \frac{2a'}{m_u m_c} + \frac{3a'}{2m_u^2} = \frac{2a'}{m_u} \left[\frac{1}{m_u} - \frac{1}{m_c} \right] = \Delta M(c)$$

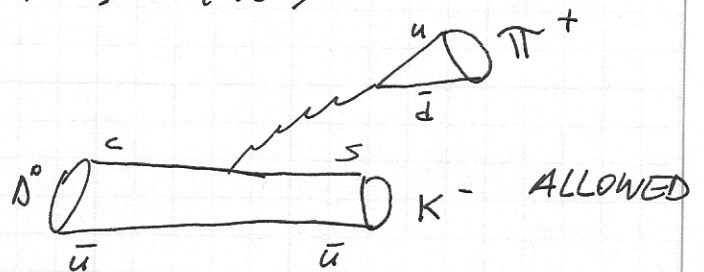
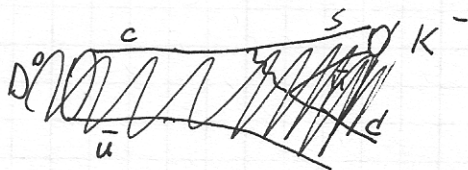
$$m(\Sigma) - m(\Lambda) = \frac{2a'}{m_u} \left[\frac{1}{m_u} - \frac{1}{m_s} \right] = \Delta M(s)$$

$$\Delta M(c) = \frac{2a'}{m_u^2} \frac{m_c - m_u}{m_c} \quad \text{and} \quad \Delta M(s) = \frac{2a'}{m_u^2} \frac{m_s - m_u}{m_s}$$

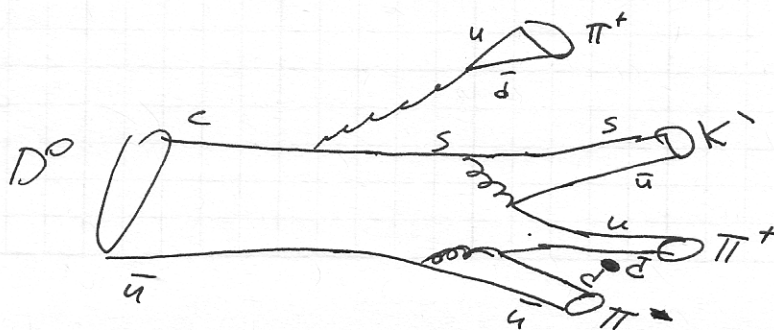
$$\Rightarrow \boxed{\Delta M(c) = \Delta M(s) \frac{m_c - m_c}{m_s - m_u} \frac{m_s}{m_c}}$$

⑨ Helzen and Martin 2.22

$$D^0 \rightarrow K^- \pi^+ \quad |c\bar{u}\rangle \rightarrow |s\bar{u}\rangle + |u\bar{d}\rangle$$



$$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^- \quad \text{ALLOWED}$$



(for example)

• $D^0 \rightarrow \pi^+ \pi^-$ no strange quark in final state

Since $c \rightarrow s$, this does not occur

(Well, it does, but it is suppressed)

$c \rightarrow d$ transition

• $D^0 \rightarrow K^+ K^-$ two strange quarks in final state -

One can come from $c \rightarrow s$ - The other one cannot come from $g \rightarrow s \bar{s}$ unless there is a strange particle - Not allowed -

(Actually this also occurs, but it is suppressed

$c \rightarrow d$ with $g \rightarrow s \bar{s}$)

• $D^0 \rightarrow K^+ \pi^-$ $c \rightarrow s$ but K^+ contains $\bar{s}$ - Forbidden -

(Actually it occurs, but it is suppressed twice $c \rightarrow W^+ d \rightarrow u \bar{s}$)

• $D^+ \rightarrow K^- \pi^+ \pi^+$ $D^+ \sim c \bar{d}$ $K^- \sim s \bar{u}$ allowed $c \rightarrow s$

• $D^+ \rightarrow K^+ \pi^+ \pi^-$ $D^+ \sim c \bar{d}$ $K^+ \sim \bar{s} u$ $c \rightarrow s$ but no $c \rightarrow \bar{s}$!

10

Ω^- has $S = -3$ (strangeness) $I = 0$ $J^P = \frac{3}{2}^+$

$B = 1$ $M = 1672 \text{ MeV}$

Strong and electromagnetic interactions conserve strangeness. For this to happen, 3 possibilities

(a) Could decay into a light baryon, say p or n

and $3K$ - $M(\Omega^-) - M(p) = 1672 \text{ MeV} - 938 \text{ MeV} = 734 \text{ MeV}$

But $M(K) = 494 \text{ MeV}$, $3M(K) > 734 \text{ MeV}$

NOT POSSIBLE

(b) Could decay into $S=-2$ baryon + 1 kaons

$S=-2$ baryon, $\Xi^{0(-)}$. $M(\Xi) = 1315 \text{ MeV}$

$$M(\Omega^-) - M(\Xi) = 1672 - 1315 = 357 \text{ MeV} \quad \text{241}$$

But $M(K) > 357 \text{ MeV}$

NOT POSSIBLE

(c) Could decay into $S=-1$ baryon + 2 kaons

$S=-1$ baryon, lightest is Λ

$$M(\Omega^-) - M(\Lambda) = 1672 - 1115 = 557 \text{ MeV}$$

But $2M(K) > 557 \text{ MeV}$

NOT POSSIBLE

Then, we are left with weak decay as the only possibility

11) Write $\pi\pi$ physical states. They must be symmetric under $1 \leftrightarrow 2$ because $J(B) = 0$ so $L(\pi\pi) = 0$

Then, we'll ~~decompose~~ decompose the physical states into isospin pieces.

Denote physical states by $|+-\rangle, |+0\rangle, |00\rangle$

$$|+-\rangle = \frac{1}{\sqrt{2}} |\pi^+ \pi^- \rangle + \frac{1}{\sqrt{2}} |\pi^- \pi^+ \rangle = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{6}} |20\rangle + \frac{1}{\sqrt{2}} |11\rangle + \frac{1}{\sqrt{3}} |00\rangle \right] + \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{6}} |20\rangle - \frac{1}{\sqrt{2}} |11\rangle + \frac{1}{\sqrt{3}} |00\rangle \right]$$

$$|+-\rangle = \frac{1}{\sqrt{3}} |20\rangle + \sqrt{\frac{2}{3}} |00\rangle$$

$$|00\rangle = |\pi^0 \pi^0\rangle$$

$$|00\rangle = \sqrt{\frac{2}{3}} |20\rangle - \frac{1}{\sqrt{3}} |00\rangle$$

$$|+0\rangle = \frac{1}{\sqrt{2}} |\pi^+ \pi^0\rangle + \frac{1}{\sqrt{2}} |\pi^0 \pi^+\rangle = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} |21\rangle + \frac{1}{\sqrt{2}} |11\rangle \right] + \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} |21\rangle - \frac{1}{\sqrt{2}} |11\rangle \right]$$

$$|+0\rangle = |21\rangle$$

$$B^+ \rightarrow \pi^+ \pi^0 \text{ is } \Delta I = \frac{3}{2}$$

Add a spurion s of isospin $|\frac{3}{2} \frac{1}{2}\rangle$ to the left side

$$A^{+0} = \langle +0 | S_{\frac{3}{2}} | B^+ s \rangle = \langle 21 | S_{\frac{3}{2}} | B^+ s \rangle$$

$$\text{But } |B^+ s\rangle = |\frac{1}{2} \frac{1}{2}\rangle |\frac{3}{2} \frac{1}{2}\rangle = \frac{\sqrt{3}}{2} |21\rangle - \frac{1}{2} |11\rangle$$

$$\Rightarrow A^{+0} = \langle 21 | S_{\frac{3}{2}} | 21 \rangle \frac{\sqrt{3}}{2} + \langle 21 | S_{\frac{3}{2}} | 11 \rangle \left(-\frac{1}{2}\right)$$

$$A^{+0} = \frac{\sqrt{3}}{2} \langle 2 \ 1 | S_{3/2} | 2 \ 1 \rangle$$

$$\boxed{A^{+0} = \frac{\sqrt{3}}{2} S_{3/2}} \leftarrow \left[\text{here } S_{3/2} \text{ is shorthand for the amplitude } \langle 2 \ I_3 | S_{3/2} | 2 \ I_3 \rangle \text{ which does not depend on } I_3 \right]$$

$B^0 \rightarrow \pi^+ \pi^-$ has a $\Delta I = \frac{3}{2}$ & $\Delta I = \frac{1}{2}$ components

Introduce two spurions ~~B^0~~ $|S\rangle = |\frac{3}{2} \ \frac{1}{2}\rangle$

$$|S'\rangle = |\frac{1}{2} \ \frac{1}{2}\rangle$$

$$A^{+-} = \langle + - | S_{3/2} | B^0 S \rangle + \langle + - | S_{1/2} | B^0 S' \rangle$$

$$|B^0 S\rangle = |\frac{1}{2} \ -\frac{1}{2}\rangle |\frac{3}{2} \ \frac{1}{2}\rangle = \frac{1}{\sqrt{2}} |2 \ 0\rangle + \frac{1}{\sqrt{2}} |1 \ 0\rangle$$

$$|B^0 S'\rangle = |\frac{1}{2} \ -\frac{1}{2}\rangle |\frac{1}{2} \ \frac{1}{2}\rangle = \frac{1}{\sqrt{2}} |1 \ 0\rangle + \frac{1}{\sqrt{2}} |0 \ 0\rangle$$

Since $|+-\rangle$ only has $I=0$ & $I=2$ components, this becomes (using $|+-\rangle = \frac{1}{\sqrt{3}} |2 \ 0\rangle + \sqrt{\frac{2}{3}} |0 \ 0\rangle$)

$$A^{+-} = \frac{1}{\sqrt{6}} \langle 2 \ 0 | S_{3/2} | 2 \ 0 \rangle + \frac{1}{\sqrt{3}} \langle 0 \ 0 | S_{1/2} | 0 \ 0 \rangle$$

$$\boxed{A^{+-} = \frac{1}{\sqrt{6}} S_{3/2} + \frac{1}{\sqrt{3}} S_{1/2}}$$

$B^0 \rightarrow \pi^0 \pi^0$ has $\Delta I = \frac{3}{2}$ & $\Delta I = \frac{1}{2}$ components

$$A^{00} = \langle \pi^0 \pi^0 | S_{3/2} | B^0 S \rangle + \langle \pi^0 \pi^0 | S_{1/2} | B^0 S' \rangle$$

~~And since~~ We have already worked out $|B^0_s\rangle$ & $|B^0_{s'}\rangle$

Also, $|\pi^0\pi^0\rangle = \frac{\sqrt{2}}{2} |2\ 0\rangle - \frac{1}{\sqrt{3}} |0\ 0\rangle$

$$\Rightarrow A^{00} = \frac{\sqrt{2}}{2} \frac{1}{\sqrt{2}} \langle 2\ 0 | S_{3/2} | 2\ 0 \rangle - \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} \langle 0\ 0 | S_{1/2} | 0\ 0 \rangle$$

$$A^{00} = \frac{1}{\sqrt{3}} S_{3/2} - \frac{1}{\sqrt{6}} S_{1/2}$$

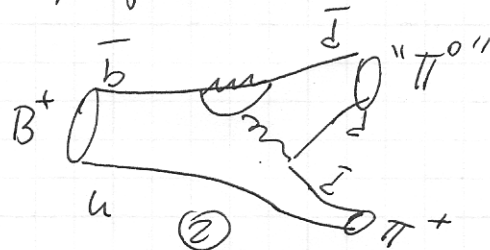
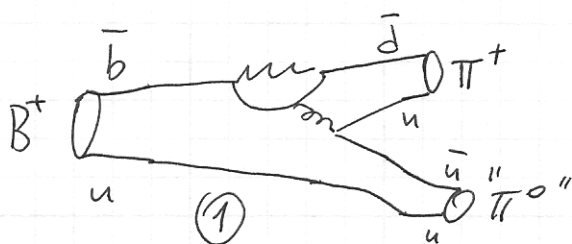
Now verify $\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}$

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{6}} S_{3/2} + \frac{1}{\sqrt{3}} S_{1/2} \right) + \frac{1}{\sqrt{3}} S_{3/2} - \frac{1}{\sqrt{6}} S_{1/2}$$

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = \frac{1}{\sqrt{12}} S_{3/2} + \frac{1}{\sqrt{3}} S_{3/2} = \frac{\sqrt{3}}{2} S_{3/2} = A^{+0} \checkmark$$

Since $\frac{1}{\sqrt{12}} + \frac{1}{\sqrt{3}} = \frac{1}{2\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

BTW Possible $B \rightarrow \pi^+ \pi^0$ penguins are



But diagram (1) $B^+ \rightarrow \pi^+ (u\bar{u})$

(2) $B^+ \rightarrow \pi^+ (d\bar{d})$

Summing them: $B^+ \rightarrow \pi^+ (u\bar{u} + d\bar{d})$

But the π^0 is $\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$ not $\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$